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LETTER TO THE EDITOR

Geometrical arguments against the Alexander–Orbach conjecture for lattice animals and diffusion limited aggregates

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Abstract. Geometrical arguments are given which suggest that the Alexander–Orbach conjecture does not hold for lattice animals in $d = 2$ and for the diffusion-limited aggregates for large dimensions.

The Alexander–Orbach (AO) (Alexander and Orbach 1982) conjecture has recently received much attention for its intriguing relation between static and dynamic exponents. This conjecture based on numerical evidence was originally made for the percolating cluster. Imagine a random walk on a percolating cluster. The fractal dimensionality of the walk d_w is defined by $r^{d_w} \sim t$ where t is the time required for a RMS displacement r. If d_f is the fractal dimension of the percolating cluster AO conjecture states that the 'fracton' dimension d_s given by $d_s = 2d_f/d_w$ has its mean field value $d_s = \frac{4}{3}$ for any value of the Eucledian dimensionality d.

An argument in favour of this conjecture was given by Rammal and Toulouse (1983) and in a more elaborate way by Leyvraz and Stanley (1983). Numerical evidence (Pandey and Stauffer 1983, Havlin and Ben-Avraham 1983, Ben-Avraham and Havlin 1983) also seems to support this conjecture. On the contrary, using the result of Wallace and Young (1978) that the resistivity exponent $\tilde{z} = 1$ to all orders in ε, Harris and Lubensky (1983) recently argued that AO should fail for percolation. However, Coniglio (1983a) and Grest (1983) have recently questioned the validity of the ε-expansion for the resistivity exponent.

More recently the AO conjecture has been tested numerically in other systems such as the diffusion limited aggregates (Meakin and Stanley 1983) and random lattice animals (Sahimi 1983, Wilke *et* 1983, Gould and Kohin 1983) in both two and three dimensions. The data have been found to be consistent with AO.

In this letter we will give a simple geometrical argument to show that AO cannot hold exactly for lattice animals in $d = 2$, and for diffusion limited aggregate for higher dimensions. We will also provide arguments to show that for random percolation and lattice animals AO conjecture is expected to hold to a good approximation in higher

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dimensions. In contrast, we show that AO conjecture holds better in *lower* dimensions for diffusion limited aggregates.

We first express the AO conjecture in a different way (Stanley and Coniglio 1984). It is well known (Gefen *et al* 1983, Havlin and Ben-Avraham 1983) that from the Einstein relation between the conductivity and the diffusion constant, using scaling arguments one can relate the conductivity exponent $\tilde{t} = d - 2 + \tilde{z}$ to the fractal dimension d_f and d_w via

$$\tilde{z} = d_w - d_f \tag{1}$$

where $\tilde{z}$ is the resistivity exponent describing the divergence of the resistance R between two points separated by a distance of the order of the connectedness length ξ: $R \sim \xi^{\tilde{z}}$. In the nodes, links and blobs model (Coniglio 1981) this would be the resistance between the nodes. From (1) it follows that the AO conjecture $d_w/d_f = \frac{3}{2}$ is equivalent to

$$\tilde{z} = \tfrac{1}{2} d_f. \tag{2}$$

In the lattice animal problem it is well established that loops are irrelevant (Lubensky and Isaacson 1979, Family 1980, 1982, Gaunt *et al* 1982, Coniglio 1983b). Therefore a lattice animal on a large scale can be viewed as a branched fractal without loops (figure 1). The resistance R between two points separated by a distance of the order of ξ clearly satisfies the inequality $R > \xi$ since the end-to-end distance is smaller than any other length. Since $R \sim \xi^{\tilde{z}}$ it follows that $\tilde{z} \geq 1$ which contradicts AO conjecture (2). In fact for $d = 2$, $d_f < 2$ (Family 1980), hence $\tilde{z} = \frac{1}{2} d_f < 1$. Note that the same argument does not apply to percolation since loops are relevant and therefore R may be less than ξ (Coniglio 1981).

For diffusion limited aggregates (Witten and Sander 1981) it is generally believed that $d_f \sim d$ for large d (Muthukumar 1983, Witten and Sander 1983). On the other hand in each branch the number of sites is expected to be proportional to ξ^2 (figure 2). Therefore for high d, $\tilde{z} = 2$ in contradiction with AO conjecture (2).

We now give an argument to show that both for percolation and lattice animals even if AO does not hold exactly, it is expected to be verified to a good approximation. For d larger than the upper critical dimensionality d_c the backbone is made of singly connected bonds which coincide with the resistance R as in figure 1. From the backbone

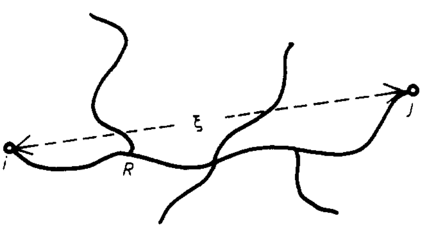

Figure 1. A lattice animal configuration. The resistance between i and j, R, is larger than ξ.

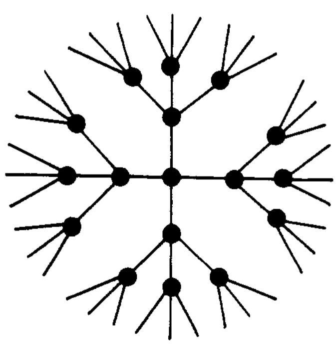

Figure 2. Diffusion limited aggregation on the Cayley tree (corresponding to large values of d). All the incoming particles occupy all the available sites giving a fractal dimensionality $d_f = d$ (i.e. no screening).

bonds other chains emanate which are dangling ends. Above d_c the excluded volume effect is absent and the critical exponent stick at their mean field value $\tilde{z}=2$, $d_f=4$ satisfying (2). A simple physical argument to show how (2) is verified above d_c is the following. Let w be the probability that from a given site on the backbone emanate a dead end chain. Then, wR is the number of dead end chains. Since each chain is roughly similar to the backbone chain we have $s^* \sim R^2$ where $s^* \sim \xi^{d_f}$ is the mass of the incipient infinite cluster. Since $R \sim \xi^{\tilde{z}}$ it follows (2). Below d_c the excluded volume effect is present and the percolation backbone is a more complex structure made of links and blobs. However, one would still expect relation (2) to hold to a good approximation due to the branched structure.

For non-branched fractals such as the backbone of the percolating cluster, the one-dimensional case and the Sierpinski gasket, the above argument shows that AO is very far from being satisfied.

In conclusion we have given a simple argument which shows that the Alexander–Orbach conjecture fails for random animals in $d=2$ and for diffusion limited aggregates for larger values of d. Whether it holds for percolation, the present consideration cannot exclude. However, if AO does not hold for some value of d this is expected to be for $d=2$. Therefore, it is at $d=2$ where the numerical efforts should be concentrated to prove or disprove AO.

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